Consider the vectors $\vec{f} = 3\vec{k} - 3\vec{i}$ and $\vec{g} = 2\vec{i} - \vec{j} - \vec{k}$. $\langle -3, 0, 3 \rangle$ $\langle 2, -1, -1 \rangle$

SCORE: ____ / 7 PTS

ALL ITEMS ON ALL QUESTIONS
(1) POINT
UNLESS OTHERWISE NOTED

[a] If $\vec{e} = 7\vec{i} + c\vec{j} - 5\vec{k}$ is perpendicular to $\vec{g}$, find the value of c.

$\vec{e} \cdot \vec{g} = 14 - c + 5 = 0$

$c = 19$

[b] Find the angle between $\vec{f}$ and $\vec{g}$. (Your answer should be in radians.)

$\cos^{-1} \frac{\vec{f} \cdot \vec{g}}{\|\vec{f}\| \|\vec{g}\|} = \cos^{-1} \frac{-6-3}{3\sqrt{2} \cdot \sqrt{6}} = \cos^{-1} \frac{-9}{6\sqrt{3}} = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$

Find the center and radius of the sphere $x^2 + y^2 + z^2 - 4x + 6y + 10z + 29 = 0$.

SCORE: ____ / 3 PTS

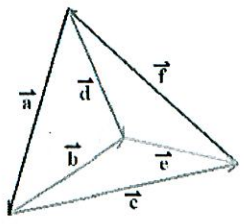
$x^2 - 4x + 4 + y^2 + 6y + 9 + z^2 + 10z + 25 = -29 + 4 + 9 + 25$

$(x-2)^2 + (y+3)^2 + (z+5)^2 = 9$

CENTER $(2, -3, -5)$ (1/2) RADIUS 3 (1/2)

Write vector $\vec{e}$ in terms of vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ in the diagram on the left.

SCORE: ____ / 2 PTS



$\vec{e} = -\vec{b} + \vec{c} = \vec{c} - \vec{b}$

Let P be the point $(-5, -2, 3)$. Let Q be the point $(3, 2, -1)$. Let R be the point $(-3, 4, -2)$. **SCORE: _____ / 18 PTS**

Let $\vec{w}$ be the vector with initial point P and terminal point R.

[a] In which octant is Q ?

$O_{1+4} = O_5$

[b] Find a unit vector in the opposite direction as $\vec{w}$.

$\vec{w} = \overrightarrow{PR} = \langle 2, 6, -5 \rangle$

$-\frac{1}{\|\vec{w}\|}\vec{w} = -\frac{1}{\sqrt{65}}\langle 2, 6, -5 \rangle = \left\langle -\frac{2\sqrt{65}}{65}, -\frac{6\sqrt{65}}{65}, \frac{5\sqrt{65}}{65} \right\rangle$

(2)

[c] If $\|\vec{v}\| = 3$, and the angle between $\vec{w}$ and $\vec{v}$ is $\frac{5\pi}{6}$ radians, find $\vec{w} \cdot \vec{v}$.

$\|\vec{w}\| \|\vec{v}\| \cos\theta = \sqrt{65} \cdot 3 \cdot -\frac{\sqrt{3}}{2} = -\frac{3\sqrt{195}}{2}$

(2)

[d] Find $\langle -1, 2, -1 \rangle \times \vec{w}$.

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 2 & -1 \\ 2 & 6 & -5 \end{vmatrix} \begin{matrix} \vec{i} & \vec{j} \\ -1 & 2 \\ 2 & 6 \end{matrix} = \begin{matrix} -10\vec{i} - 2\vec{j} - 6\vec{k} \\ +6\vec{i} - 5\vec{j} - 4\vec{k} \end{matrix} = \langle -4, -7, -10 \rangle$$

(3)

CHECK: $\langle -4, -7, -10 \rangle \cdot \langle -1, 2, -1 \rangle = 4 - 14 + 10 = 0$ ✓

$\langle -4, -7, -10 \rangle \cdot \langle 2, 6, -5 \rangle = -8 - 42 + 50 = 0$

[e] Find the equation of the sphere with P and Q as endpoints of a diameter.

CENTER = MIDPOINT = $(-1, 0, 1)$

RADIUS = $\frac{1}{2}$ DISTANCE = $\frac{1}{2}\sqrt{8^2 + 4^2 + 4^2} = 2\sqrt{6}$

$(x+1)^2 + y^2 + (z-1)^2 = 24$

(2)